Prove that $\frac{1}{1\times5}+\frac{1}{5\times9}+\frac{1}{9\times13}+\cdots+\frac{1}{(4n-3)\times(4n+1)}=\frac{n}{4n+1}$ for all integers $n\geq1$ **SCORE: _____ / 10 PTS**

**by mathematical induction, showing all steps demonstrated in lecture.**

BASIS STEP:

If $n=1$, $\boxed{\frac{1}{1\times5}=\frac{1}{5} \qquad \frac{1}{4(1)+1}=\frac{1}{5}}$ 1

INDUCTIVE STEP:

1 $\boxed{\text{Assume } \frac{1}{1\times5}+\frac{1}{5\times9}+\frac{1}{9\times13}+\cdots+\frac{1}{(4k-3)\times(4k+1)}=\frac{k}{4k+1}}$ $\boxed{\text{for some arbitrary but particular integer } k}$ 1

2 $$\boxed{\frac{1}{1\times5}+\frac{1}{5\times9}+\frac{1}{9\times13}+\cdots+\underbrace{\frac{1}{(4k-3)\times(4k+1)}+\frac{1}{(4k+1)\times(4k+5)}}}$$

1 $$\boxed{=\frac{k}{4k+1}+\frac{1}{(4k+1)(4k+5)}}$$

FIRST & LAST 2 TERMS MUST BOTH BE SHOWN

$$=\frac{k(4k+5)+1}{(4k+1)(4k+5)}$$

1 $$\boxed{=\frac{4k^2+5k+1}{(4k+1)(4k+5)}}$$

1 $$\boxed{=\frac{(4k+1)(k+1)}{(4k+1)(4k+5)}}$$

$$=\frac{k+1}{4k+5}$$

1 $$\boxed{=\frac{k+1}{4(k+1)+1}}$$

1 $\boxed{\text{By mathematical induction, } \frac{1}{1\times5}+\frac{1}{5\times9}+\frac{1}{9\times13}+\cdots+\frac{1}{(4n-3)\times(4n+1)}=\frac{n}{4n+1} \text{ for all integers } n\geq1}$

Evaluate $\sum_{k=1}^{200}(8k-3k^2)$. **Your final answer must be a number (not involving arithmetic operations).** SCORE: _____ / 4 PTS

$$= \underbrace{8\sum_{k=1}^{200} k - 3\sum_{k=1}^{200} k^2}$$ 1

$$= 8\cdot\underbrace{\tfrac{1}{2}(200)(201)}-3\cdot\underbrace{\tfrac{1}{6}(200)(201)(401)}$$ 1 1

$$= \underbrace{-7899300}$$ 1

Find the $7^{th}$ term of $(11b-8g)^{26}$. **Your final coefficient may be in factored form as shown in lecture.** SCORE: _____ / 5 PTS

$_{26}C_6 (11b)^{26-6}(-8g)^6$

$= \frac{26!}{6!\,20!}(11b)^{20}(-8g)^6$ (1/2) (1/2)

PLUS (1/2) IF YOUR FINAL ANSWER IS POSITIVE

$= \frac{26\cdot 25\cdot 24\cdot 23\cdot 22\cdot 21\cdot 20!}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 20!}\, 11^{20}\, 8^6\, b^{20} g^6$ (1)

$= 26\cdot 5\cdot 23\cdot 11\cdot 7\cdot 11^{20}\cdot 8^6 b^{20} g^6 = 230230\cdot 11^{20}\cdot 8^6\, b^{20} g^6$ (1/2) (1/2) (1/2) (1)

If $f(x)=x^5$, expand and **completely simplify** the difference quotient $\frac{f(x+h)-f(x)}{h}$. SCORE: _____ / 5 PTS

$\frac{(x+h)^5 - x^5}{h} = \frac{\cancel{x^5} + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5 - \cancel{x^5}}{h}$ (3)

$= 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4$ (2)

Expand and **completely simplify** the complex number $(3-2i)^4$. SCORE: _____ / 6 PTS

$1(3)^4(-2i)^0 + 4(3)^3(-2i)^1 + 6(3)^2(-2i)^2 + 4(3)^1(-2i)^3 + 1(3)^0(-2i)^4$ (1/2) (1/2) (1/2)

$= 81 + 4(27)(-2i) + 6(9)(-4) + 4(3)(8i) + 16$ (1/2) (1/2)

$= 81 - 216i - 216 + 96i + 16$ (1/2) (1/2) (1)

$= -119 - 120i$ (1 1/2)